Dark Energy: relating the evolution of the universe from the past to the future

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Abstract

Using the evolution history of the universe, one can make constraint on the parameter space of dynamic dark energy models. We discuss two different parameterized dark energy models. Our results further restrict the combined constraints obtained from supernova and WMAP observations. From the allowed parameter space, it is found that our universe will experience an eternal acceleration. We also estimate the bound on the physically relevant regions both in the re-inflationary and inflationary phases.

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Numerous and complementary cosmological observations made in the last decade suggest that the expansion of our universe is now accelerating (re-inflation) [1]. This may indicate that our universe contains dark energy with equation of state $w_{de} < -\frac{1}{3}$, making up as much as 70% of the critical energy density. The usual suspects for dark energy are cosmological constant with constant equation of state $w_{de} = -1$ or exotic fields with time dependent equation of state [2].

In addition to telling us contents of our universe, acceleration of the cosmic expansion has also profound implications for dynamics and physics on both very high energy scale in the past and low energy scale in the future. Using the characteristic length scale of expansion, the Hubble radius H^{-1}, together with the energy density in terms of the expansion factor, we can distinguish our universe into three different phases starting from inflationary expansion (inflation), following by radiation and matter dominated phase in standard big bang cosmology and finally entering the re-inflationary era governed by the dark energy. For an eternally accelerating third phase driven by the dark energy, the event horizon surrounding any observer will limit the number of e-folds of the re-inflation that will be observable in our universe. More interestingly, it was found that when a universe is at the moment of the transition to the eternally re-inflationary phase, it contains the most information about the first phase inflationary perturbations. This could impose a bound on the physically relevant duration of the first phase inflation. Considering the dark energy as the cosmological constant, the concept of investigating the dynamic ranges during inflationary and re-inflationary phases has been discussed in [3, 4]. Besides, the phenomenology for the expansion dynamics in the second phase of the universe evolution influenced by the dark energy has also been examined in [5]. It was argued that the dark energy serves as a thread of three phases of the evolution of our universe, relating the past to the future.

In this paper, we are going to generalize previous studies and examine the dynamical implications on the past and future expansion behaviors of our universe due to the dark energy with time-dependent equation of state. The evolving dark energy is an alternative model to the cosmological constant, which affects the features of the temperature anisotropies in the cosmic microwave background radiation and influences a lot on the small l CMB spectrum [6, 7]. In addition to its relevance to the early universe evolution, we will show that the behavior of the variation of equation of state for dark energy determines the fate of our universe. Considering the evolution of energy density with the expansion of the universe, we will present that the history of the universe expansion on the contrary also puts constraints on any possible variation of equation of state for dark energy. The expansion of the universe and the evolution of dark energy are closely co-related.

The expansion of the background universe is described by

$$H^2(a) = H_0^2 \left[\Omega_r^0 a^{-4} + \Omega_m^0 a^{-3} + \Omega_d^0 f(a)\right], \tag{1}$$

where $f(a) = \exp\left[3\int_a^1 \frac{1+w_{de}(a')}{a'}\mathrm{d}a'\right]$, Ω_r, Ω_m are dimensionless radiation and matter densities and Ω_d is the dark energy density. This Friedmann equation can be used to give pictures of the second and third phases of the evolution of the universe. For the first inflationary phase, we simply assume that the Hubble parameter remains a constant H_I. And we further assume that this phase finished with a perfectly efficient reheating so that the radiation era started just at the end of the first phase with the scale factor $a_{\rm end}$.

The effective total equation of state of the universe can be calculated by [5, 8]

$$\begin{aligned} w_{tot}(a) &= -1 - \frac{1}{3}\frac{\mathrm{d}\ln(H^2/H_0^2)}{\mathrm{d}\ln a}, \\ &= \frac{\Omega_r^0 a^{-4} + 3\Omega_d^0 f(a) w_{de}(a)}{3\left[\Omega_r^0 a^{-4} + \Omega_m^0 a^{-3} + \Omega_d^0 f(a)\right]}. \end{aligned} \tag{2}$$

In the radiation era, $w_{tot} = \frac{1}{3}$. At present, since we are already in the re-inflationary era according to observations, $w_{tot} < -\frac{1}{3}$. The re-inflation started at $z_* = 0.46 \pm 0.13$ at 1σ [13], which requires $q = -\frac{a}{H}\frac{\mathrm{d}H}{\mathrm{d}a} - 1$ crossing 0 and puts $w_{tot} = -\frac{1}{3}$ at $a_* = \frac{1}{1+z_*}$. All these historical moments of the universe expansion could give constraints on the dark energy evolution. We will show that combining the constraints from the expansion history with supernova type Ia and WMAP observations, we can severely restrict possible parameter space in the time-dependent dark energy models.

In the far future, it is easy to see from Eq. (2) that $w_{tot} \to w_{de}$. Thus the asymptotic behavior of the evolving dark energy at large scale will determine the fate of the evolution of our universe. The re-inflationary phase could keep on forever or give way to the decelerated expansion provided that at large scale the time-dependent equation of state approximates to $w_{de} < -\frac{1}{3}$ or $w_{de} > -\frac{1}{3}$ respectively.

If the universe ends with a decelerated expansion, a patient observer could get as much information from CMB spectrum of our universe as he/she observes. However, while the re-inflationary phase lasts forever, the observer will always be surrounded by a cosmological event horizon. Spacetimes with event horizons contain Hawking particles. As the universe acceleratingly expands, the wavelength of the CMB photons will be redshifted rapidly and CMB temperature will drop below the Hawking temperature. After this the CMB information will be completely immersed in the cosmological Hawking radiation. We illustrated this idea in Fig 1. Thus if the universe starts to

accelerate forever in the third phase, there will be limited region of the universe accessible to observers. For the eternally accelerating phase driven by the cosmological constant, the investigation on the cutoff scale in the observable region was carried out in [3, 4]. Employing the combined constraints on the equation of state of dark energy, we will estimate the finite physical relevant region in the third phase for dynamic dark energy models.

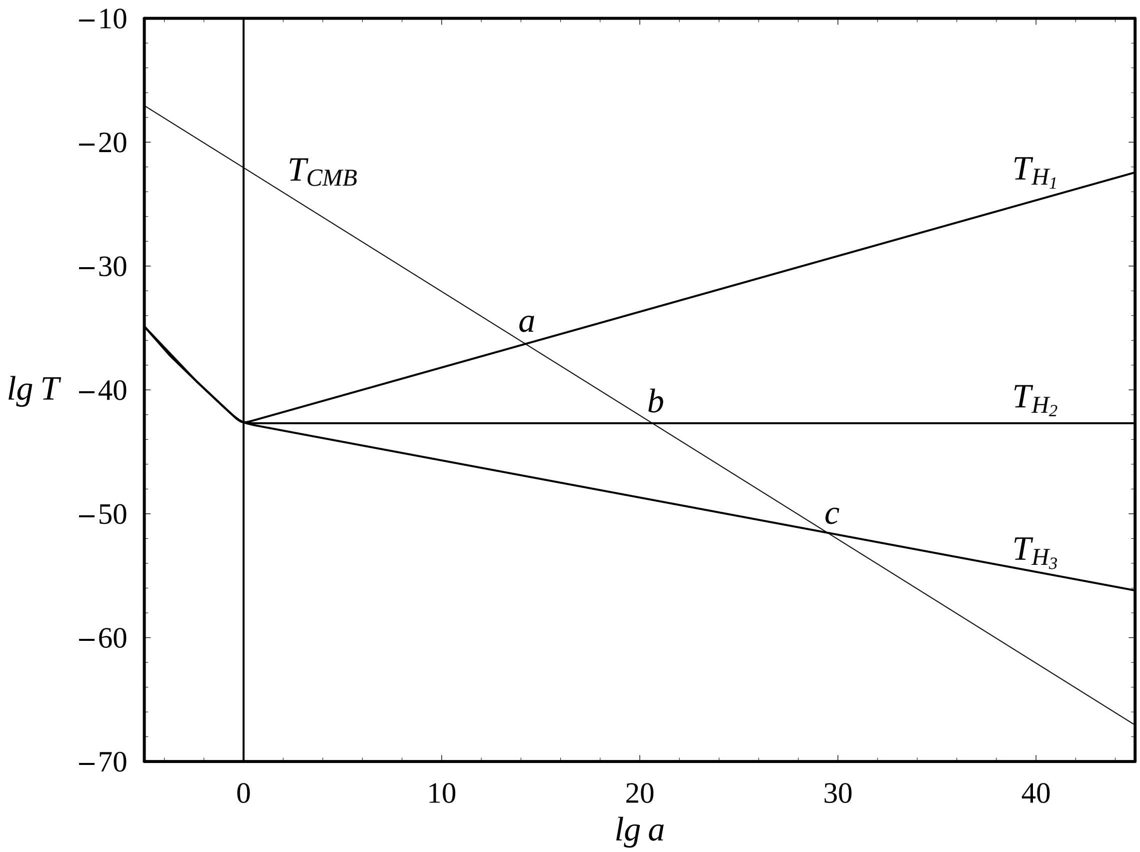

Figure 1: Evolution of the temperature of the CMB T_{CMB} and Hawking particles T_H. Points a, b and c represent the time that CMB information is immersed in Hawking radiation for $w_{de} < -1$, $w_{de} = -1$ and $w_{de} > -1$ respectively .

Similar bounds on the physically relevant duration of the first phase inflation also exist. It is known that the inflationary perturbations could act as seeds of structure formation after they re-enter the Hubble horizon. The wavelength of the quantum fluctuations stretches as $\lambda \sim a(t)$, while after the first phase, the Hubble horizon of the universe grows linearly in time, which is much faster than the stretch of the wavelength scale of the fluctuations. Hence normally the perturbations will reenter the Hubble radius sooner or later after their eviction from it if there was no re-inflation. If the universe enters the third phase with forever acceleration, the Hubble

radius will be flattered (or shrunk, depending on the equation of state for dark energy), grows even slower than the perturbation wavelength. Some of the perturbations will never reenter the Hubble horizon. The later the fluctuations reenter the Hubble radius, the earlier they exit from the Hubble horizon during the first phase. The last moment for the fluctuations generated during inflation to reenter the Hubble horizon as the universe enters the third phase is exactly the moment when the re-inflation starts. Perturbations crossing the Hubble horizon much earlier during inflation will never reenter the Hubble horizon in the re-inflation phase. Employing the criterion that we need the perturbation to reenter the Hubble horizon, we can get the astrophysical relevant bound on the length scale produced during the inflation. Clear picture of this idea is shown in Fig 2 [3]. Considering the third phase of the universe evolution being driven by the dynamic dark energy, we will find that the starting moment of the re-inflation will be shifted from that of assuming the third phase being de-Sitter phase [3]. The shift of the starting point of the re-inflation will change the bound on the duration of inflation obtained in [3]. This will be discussed in some detail for different dynamical dark energy models below.

In our following discussion, we will choose two time-dependent dark energy models with different parametrizations:

$$w_{de}^{I}(a) \;=\; w_0 + w_1(1-a), \tag{3}$$

$$w_{de}^{II}(a) \;=\; w_0 + w_1(1-a)a. \tag{4}$$

These two models have been extensively discussed in various papers, for example, see [6, 9, 10, 11, 12]. For the first model, characteristic moments of the universe evolution require that during the radiation era $w_{tot} = \frac{1}{3}$ and at the present time $w_{tot} < -\frac{1}{3}$ due to the observed accelerated expansion, which gives constraints: $w_1 \leq 0.34 - w_0$ and $w_0 < -0.48$. These constraints shrink the allowed parameter space of $w_0 - w_1$ obtained from supernova observation. Using the observational result that the re-inflation started at $z_* = 0.46 \pm 0.13$[13], which is the point where the deceleration parameter $q = -\frac{a}{H}\frac{\mathrm{d}H}{\mathrm{d}a} - 1$ crossing 0, we can have further constraints on the $w_0 - w_1$ parameter space for this dynamic dark energy model. The combined constraints from the evolution history and the supernova and WMAP observations are shown in the white region in Fig 3. The result shows that adding the evolution history constraint, the variation of the equation of state of dark energy can be severely restricted.

In the far future, we know that $w_{tot} \to w_{de}$ from Eq. (2). The fate of the universe is determined by the value of w_1. For $w_1 < 0$, the re-inflationary expansion of the universe will give way to deceleration at very late time. However, if $w_1 > 0$, w_{tot} will be a very negative value at large scale,

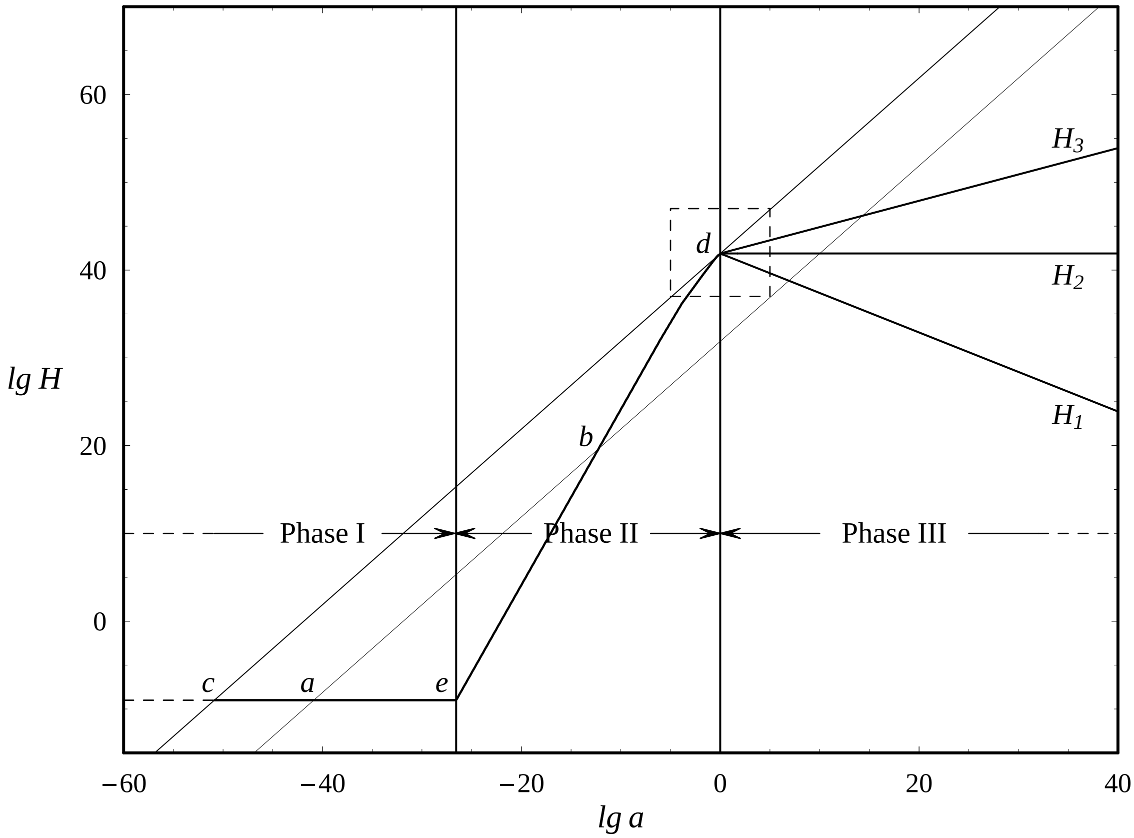

Figure 2: The evolution of our universe experiences three phases, namely inflation (line ce), radiation and matter domination (line ed) and re-inflation. Point a and b show the time perturbations generated during inflation exit and re-enter the Hubble radius. The perturbations which leave Hubble radius earlier than c will never re-enter due to the acceleration in the third phase. During re-inflation, H_1, H_2 and H_3 represent the evolution of Hubble radius in the future with $w_{de} < -1$, $w_{de} = -1$ and $w_{de} > -1$ respectively. The beginning moments of reacceleration are different due to the variations in the equation of state for dark energy.

thus the re-inflation of the universe will last forever. The combined constraints shown in Fig 3 tell us that the allowed w_1 is indeed positive leading to the eternal re-inflation of the universe in the third phase of its evolution. Using the concept explained above, the bound on the region of the universe could be accessible to observers in the future is within the range with e-folds $N_{\rm re-inf} \in [3.68, 7.91]$ by employing the allowed parameter space in Fig 3.

On the other hand, by fixing the starting moment of the re-inflation within the constraint parameter space of the evolving dark energy and using the criterion that we need the fluctuations to reenter the Hubble radius, the natural bound on the duration of the inflation with the number

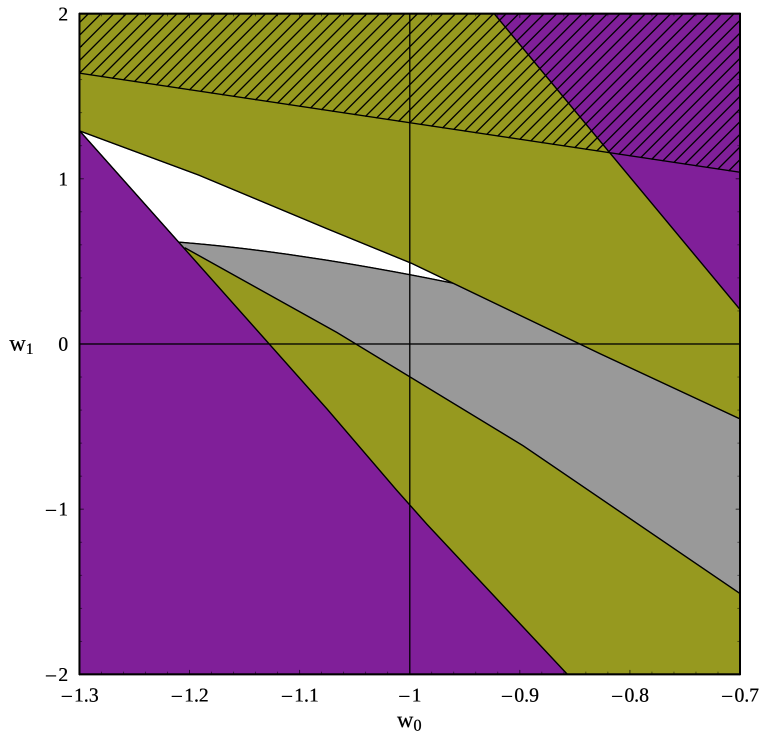

Figure 3: The purple region is excluded by supernova observations and the green part is ruled out by the WMAP observations [3]. The shaded region violets $w_{tot} = \frac{1}{3}$ at radiation era. The gray part is dropped by requiring the transition to the re-inflation phase happened at $z_* = 0.46 \pm 0.13$.

of e-folds is found to be $N_{\rm inf} \in [55.98, 56.02]$.

These studies can be extended to the second parametrization of the dynamic dark energy model. The universe evolution history requires (**a**) at the radiation era $w_{tot} = \frac{1}{3}$, (**b**) the deceleration parameter crosses 0 at a_*, (**c**) at present $w_{tot} < -\frac{1}{3}$. Especially the requirement (b) can be used to further restrict the constraints got by the supernova and WMAP observations. The combined constraint is shown in the white region in Fig 4 .

Similar to the first model, the sign of w_1 determines the fate of the universe. For $w_1 < 0$, the present re-inflation era will be followed by a deceleration expansion. However for $w_1 > 0$, the present accelerated expansion will go on forever. From the combined constraint shown in Fig 4 , we learn that our universe will experience an eternal acceleration. The bound on the number of e-folds of the future universe, which is observable, is found within the range $N_{\rm re-inf} \in [2.29, 2.87]$. In addition, using the criterion discussed above, the natural cutoff length scale in the inflation is estimated to be within the number of e-folds $N_{\rm inf} \in [55.95, 56.01]$. This number is quite near to that in the first model.

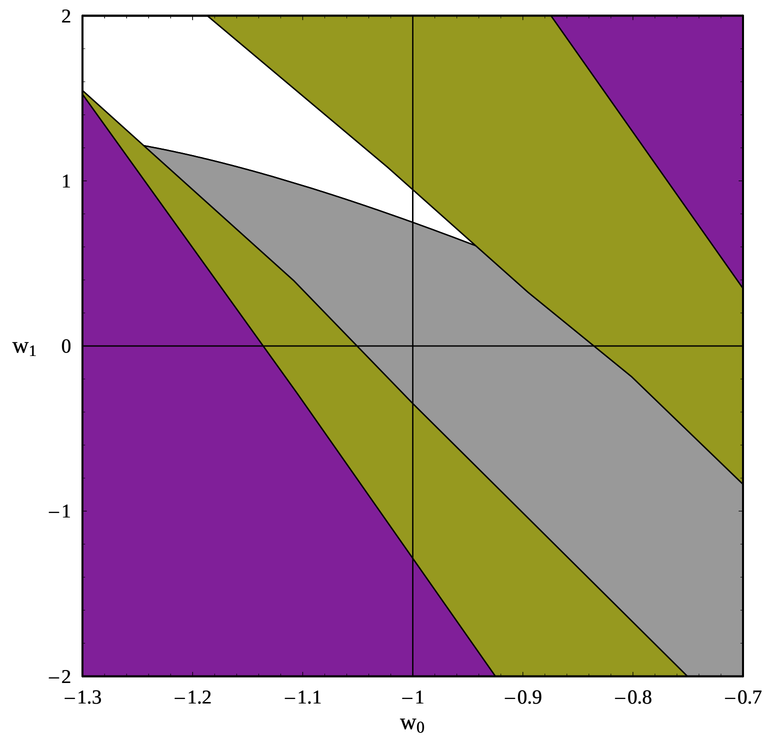

Figure 4: Similarly as explained in Fig 3, the white region left is the combined constraints resulting from both the history of the universe expansion and supernova and WMAP observations.

In summary, we investigated the close relation between the expansion history of the universe and the evolution of the dynamic dark energy. We found that some specific historical moments of the universe evolution can be used to constrain the parameter space of the evolving dark energy models, which further refines the combined constraints from supernova and WMAP observations. The behavior of the evolving dark energy can determine the fate of the universe. For two dynamic dark energy models with different parametrizations, we found that the universe will experience an eternal acceleration. In addition, the natural cutoff length scales in both the inflation and re-inflation phases were obtained, which make regions of physical relevance in both of these two phases finite.

Acknowledgments

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